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ON CALCULATING RADIATIVE FLUXES AND TEMPERATURE CHANGES IN ATMOSPHERES OF ARBITRARY STRUCTURE

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On Calculating Radiative Fluxes and Temperature Changes in Atmospheres of Arbitrary Structure

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Frank Haurwitz, Editor   Dolores Lofgren, Translator

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EDITOR'S PREFACE

The general circulation model (GCM) used by the Rand/ARPA Climate
Dynamics Project has until recently employed an expression for the
absorption of solar radiation by water vapor which was developed by
Mügge and Möller based mainly on work by Fowle. This expression also
has been similarly used in the GCMs of UCLA (Mintz-Arakawa) and the
National Center for Atmospheric Research. Since Mügge and Möller's
approach still maintains some currency and certainly has in the past
constituted a cornerstone in the treatment of the atmospheric absorp-
tion of solar radiation, it seemed desirable to make available a trans-
lation of the original article. In addition, a large portion of the
article is devoted to the graphical computation of radiative flux
and, in fact, represents the initial application of this approach
to the problem of radiative transfer. Although this is not of direct
concern to our present work, it certainly is of general historical
interest to all those dealing with the problem of radiative transfer.
For these reasons, then, the translation has been carried out. The
final translation has deviated from the literal translation only
where it has been necessary in order to clarify the meaning.

The assistance of Dr. Bernhard Haurwitz in the translation of
certain particularly troublesome passages is gratefully acknowledged.

SUMMARY

A graphical procedure is developed whereby radiative fluxes may be calculated, and thereby the temperature changes in atmospheres of arbitrary structure, taking into account the laws for the absorption of diffuse radiation with absorption coefficients that vary with wavelength. The absorption of direct solar radiation is also taken into account.

ON CALCULATING RADIATIVE FLUXES AND TEMPERATURE CHANGES
IN ATMOSPHERES OF ARBITRARY STRUCTURE

R. Mügge and F. Möller (Frankfurt/Main)

The papers by G. C. Simpson [1] and R. Mügge [2] on the radiative relationships in the atmosphere were based on greatly simplified assumptions regarding the absorption spectrum of water vapor. Quite recently, F. Albrecht [3] and O.F.T. Roberts [4] simultaneously advanced the theory of thermal radiation of atmospheric water vapor by considering the selective absorption of water vapor as established by measurements by Hettner et al., on the one hand, and also by applying the geometrical laws of radiation as developed by E. Gold [5] and R. Emden [6]. The great advance contributed by the improved theory is countered by the disadvantage that the calculation becomes extremely long and drawn out. Both Albrecht and Roberts presume average distributions of humidity and temperature in the vertical and calculate the radiative fluxes at various atmospheric levels. To this end the calculation is to be performed separately for every wavelength, since the coefficient of absorption is different at every wavelength λ, and for every λ the dependence of radiant energy on temperature is given by a different formula (Planck's law). Then the integration over all wavelengths may be performed.

It is now to be assumed that different distributions of the water vapor which absorbs and emits radiation in the atmosphere and different temperatures of the sources of radiation (water vapor and the earth's surface) yield quite different radiative flux distributions. A tropical atmosphere and a polar one, or a high-pressure air mass and a low, will exhibit great differences. The changes in the radiation that accompany changes in the structure of the atmosphere claim special interest with regard to explaining the vertical temperature profile in the stratosphere [2]. Evidently the calculations become interminable at this point, for there are, so to speak, four independent variables involved in calculating the radiation: wavelength, the atmospheric

level involved, temperature, and water vapor content. In order to
study many differently structured atmospheres without impairing the
precision of the theory, integration over all wavelengths is antici-
pated in this report. A graphical procedure is developed for deter-
mining the radiative flux at any atmospheric level from the water vapor
above and below that level and from the earth's surface. The effect
of direct solar radiation will be taken into account later.

The basic assumption is made that the coefficient of absorption
k_λ of water vapor is uniquely defined by the relative attenuation
which radiation at wavelength λ undergoes as it penetrates the quantity
of water vapor dw (measured in centimeters of precipitable water),
irrespective of the pressure or temperature of the water vapor at the
time. We use the values for k_λ given and applied by Albrecht [3].

If a quantity of water vapor w lies between an ideal[1] radiating
surface AA (Fig. 1) and a unit area B intercepting radiation from AA,
then the normally incident component of the monochromatic radiation
impinging on B at the angle of incidence z is given by

$$2\pi \cdot \frac{r \sin z \cdot rdz}{\cos z} \cdot \cos^2 z \cdot \frac{1}{r^2} \cdot e^{-\frac{k_\lambda \cdot w}{\cos z}} \cdot i_{\lambda T} \cdot d\lambda =$$

$$2\pi \cdot i_{\lambda T} d\lambda \cdot \sin z \cdot \cos z \cdot e^{-\frac{k_\lambda \cdot w}{\cos z}} \cdot dz.$$
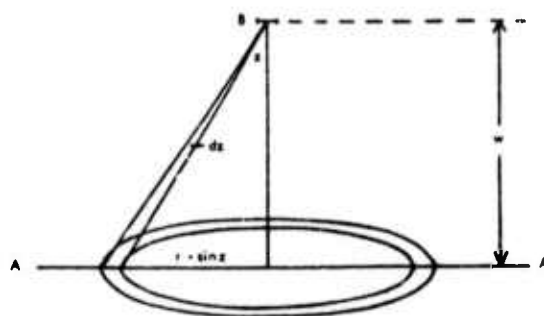

Figure I

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[1] Black body.

Here $i_{\lambda T}$ is the monochromatic radiation emitted by a unit area of the ideal radiating surface per unit solid angle:

$$i_{\lambda T}d\lambda = 2c_1\lambda^{-5}\left(e^{\frac{c_2}{\lambda T}} - 1\right)^{-1}d\lambda; \qquad c_1 = 8.420 \text{ g cal} \cdot \text{cm}^2 \cdot \text{min}^{-1};$$

$$c_2 = 1.43 \text{ cm} \cdot \text{deg}.$$

Integration over z yields the radiation from an infinitely large surface AA:

$$S^S_{\lambda T}(w)d\lambda = 2\pi i_{\lambda T}d\lambda \int_0^{\pi/2} \sin z \cos z \; e^{-\frac{k_\lambda w}{\cos z}} dz.$$

With $1/\cos z = \xi$, this becomes

$$2\pi i_{\lambda T}d\lambda \int_1^\infty e^{-k_\lambda w\xi} \cdot \xi^{-3}d\xi = 2\pi i_{\lambda T}d\lambda \frac{1}{2}\left(e^{-k_\lambda w}(1 - k_\lambda w) - k_\lambda^2 w^2 E_i(-k_\lambda w)\right)$$

$$= 2\pi i_{\lambda T}d\lambda H_3(k_\lambda w) \tag{1}$$

in the convenient notation of Gold. Since r and thus all distances drop out, the irradiated quantity of water vapor w (in cm of precipitable water) is henceforth used as a measure of distance. This should be calculated as increasing from the surface B, whether upward or downward. The radiation of an infinitely large surface through an absorbing layer of gas varies, then, not according to an exponential function, but in accordance with the function H_3. The latter can easily be tabulated in accordance with the above development and applied to subsequent calculation. The curve for this is given in Fig. 2 along with its logarithmic derivative, the latter providing a particularly clear illustration of the difference between this expression and an exponential function.
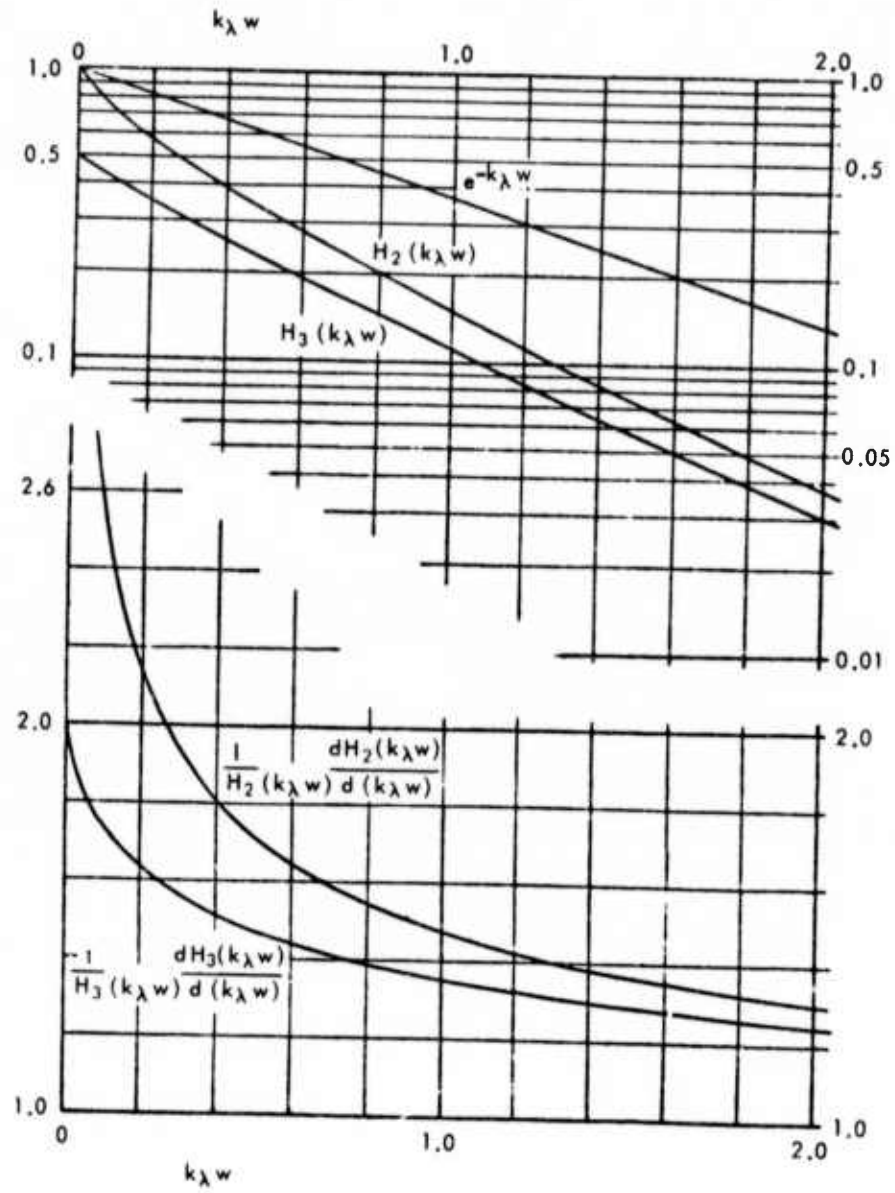
Figure 2

−5−

The total radiation in all wavelengths emitted by the black-body surface is then given by

$$S_T^S(w) = 2\pi \int_0^\infty i_{\lambda T} H_3(k_\lambda w) d\lambda. \tag{2}$$

This integration is performed graphically for temperatures between +40°C and −60°C and for water vapor in amounts up to 5 cm of precipitable water. Figure 3 gives the curve for the integration and clearly
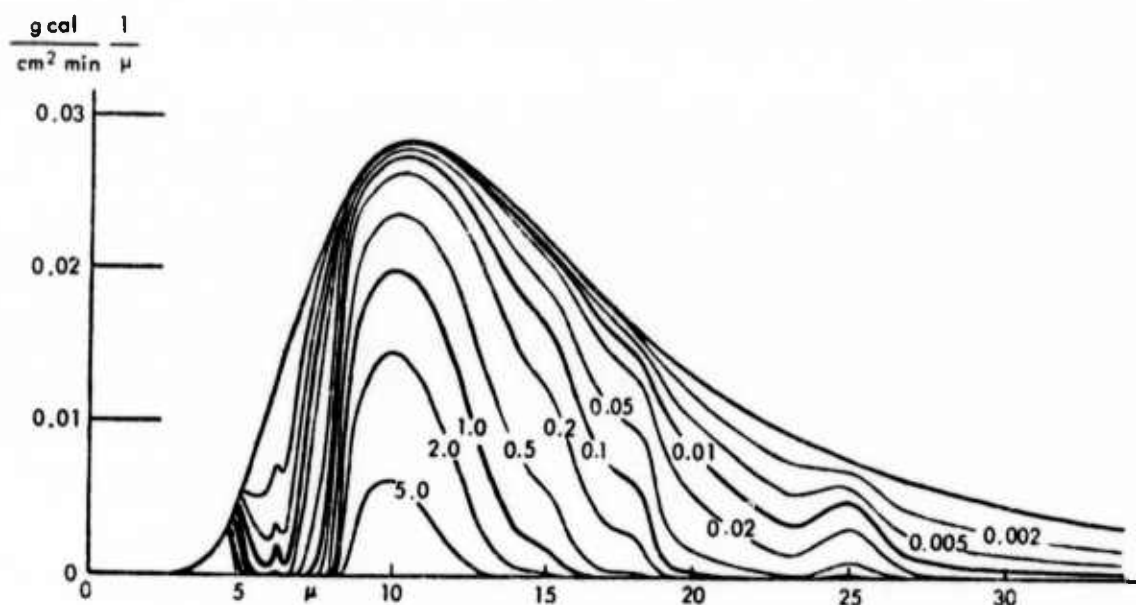
Fig. 3 -- Ideal radiation at 273° through an absorbent layer of water vapor w

shows the gradual decrease in intensity, first in the wavelengths of strong absorption, and, after the energy has completely disappeared there, in the regions with smaller k_λ. Figure 4 (left) shows S_{313}^S as a function of w, where the extremely rapid drop in intensity with very small quantities of water vapor and the slow drop with large quantities are particularly striking. This occurs because the values of the absorption coefficient vary between 0.2 and 250 and come into effect in succession, depending on their magnitude. It is obvious that no approximation of this curve is possible by means of an exponential function (gray absorption).
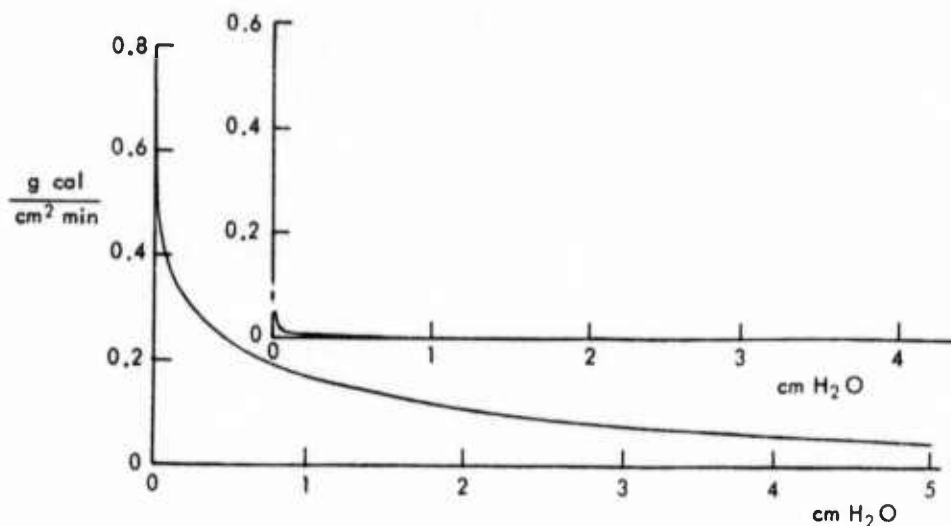
Fig. 4 -- Absorption of ideal radiation and natural
radiation by water vapor

The radiation emitted by the water vapor can itself be calculated
as follows: If AA in Fig. 1 is not considered to be an ideal radiating
surface, but rather an infinitely thin layer of water vapor dw with a
temperature T, then one may determine first of all the radiation from
this layer transmitted through the absorbent layer w resting above it.
Let the layer w be regarded only as absorptive, not as radiating.
The radiation of the elementary layer dw is defined in terms of its
absorption, according to Kirchhoff's law. A vertically incident beam
is attenuated by the factor $k_\lambda dw$, but a beam incident obliquely from
below at an angle z is attenuated by the factor $\left(k_\lambda \dfrac{dw}{\cos z}\right)$, since the
thickness of the layer penetrated is greater than in the case of vertical
incidence. Thus, the emission at angle z is given by $i_{\lambda T} d\lambda k_\lambda \dfrac{dw}{\cos z}$,
which appears in place of $i_{\lambda T} d\lambda$ in Eq. (1). Then the same calcu-
lation as above is performed:

$$S_{\lambda T}^{dw}(w)d\lambda = 2\pi i_{\lambda T} d\lambda k_\lambda dw \int_1^\infty e^{-k_\lambda w\xi} \xi^{-2} d\xi$$

$$= 2\pi i_{\lambda T} d\lambda k_\lambda dw \left(e^{-k_\lambda w} + k_\lambda w E_1(-k_\lambda w)\right) = 2\pi i_{\lambda T} d\lambda k_\lambda dw H_2(k_\lambda w)$$

(3)

S^{dw} is strictly speaking a differential, but this notation is adopted here for the sake of greater clarity. H_2 is even less easily approximated than H_3 by the simple exponential function (Fig. 2). It is similarly possible to tabulate it and to perform the integration over λ.

$$S_T^{dw}(w) = 2\pi dw \int_0^\infty i_{\lambda T} k_\lambda H_2(k_\lambda w) d\lambda. \qquad (4)$$

This integral was determined graphically for all temperatures of the radiating vapor layer present in the atmosphere (between +40° and -80°C) and for all reasonable amounts of the absorbing water vapor. Figure 5 shows the curve for the calculation at $T = 273°$. The uppermost curve gives the radiation before any absorption has occurred, and the others indicate the amounts as the radiation penetrates deeper into the water vapor layer where in cases of greater absorption one clearly sees a displacement to wavelengths that certainly radiate less, but are at the same time not all that greatly attenuated. Of course, the radiation
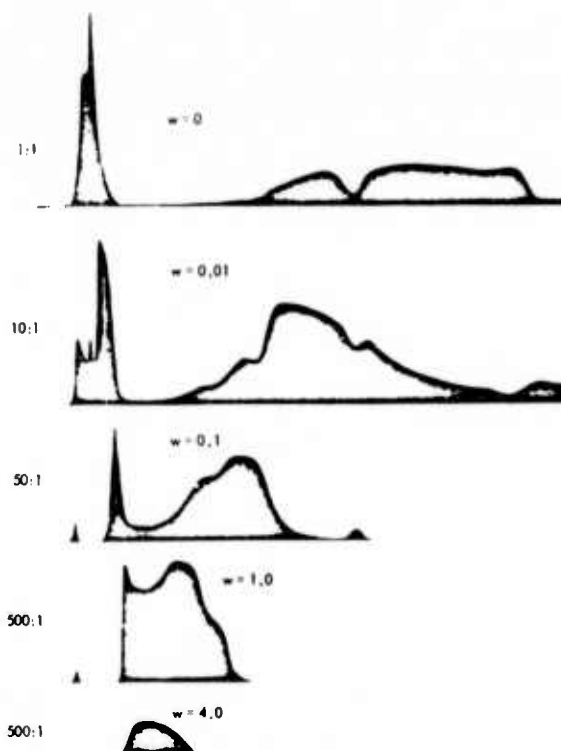

Fig. 5 -- Changes in natural radiation with the thickness of the absorbent layer (note magnification of vertical scale)

penetrating 1 cm of H_2O is already remarkably slight, and can only be represented approximately in Fig. 4 (right); still, its total energy is not negligible compared with the quantities of energy produced by nearby layers of the atmosphere because the radiating masses of these layers are less by some powers of ten.

Figure 6a gives another schematicized representation of the amount of radiation emitted by the elementary water-vapor layer dw at temperature T and transmitted through the absorbing vapor layer w. Planimetering over the area under the curve T_0 between the abscissae 0 and w gives the total amount of energy that emerges from a unit surface area of an atmosphere of finite thickness w and temperature T_0 at its boundary. If one draws a curve in the grid of T and w in 6a to indicate for each case the prevailing temperature at the "distance" w (cm of H_2O), then the area gives the radiation of an atmosphere with the given temperature distribution. Such an operation, however, cannot
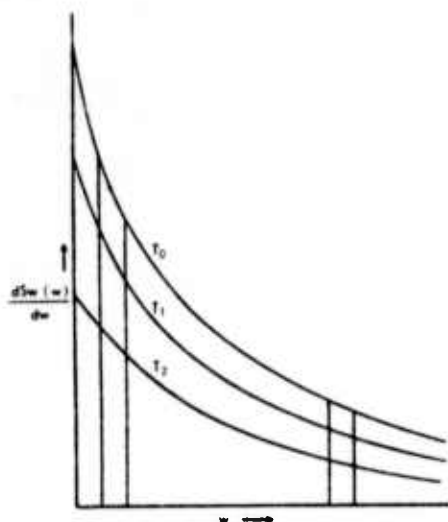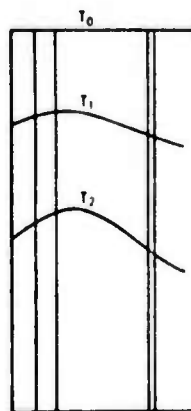

Figure 6a   Figure 6b

be performed with Fig. 4, which gives the distribution of the natural radiation. For this reason, 6a or 4 is transformed into 6b, as shown. The linear scale for w is severely compressed, so that the curve T_0 is raised while preserving the area, and also the w-scale is chosen

in such a way that the line for T_0 becomes horizontal with the ordinate equal to 1. The lifting of the other T-lines takes place perforce, since the spatial relationship of T to T_0 and to the abscissa remains the same. The w-scale is obtained computationally as follows: An isothermal atmosphere of thickness W and temperature T_0 radiates

$$S_{T_0}^W = \int_0^W S_{T_0}^{dw}(w) = 2\pi \int_0^\infty i_{\lambda T_0} d\lambda \int_0^W H_2(k_\lambda w) k_\lambda dw, \qquad (5)$$

and since the ordinate must equal 1, this quantity gives the scale for w on the abscissa.

From the definition for H_2 and H_3 we now have

$$\frac{d}{d(kw)} H_3(kw) = \frac{d}{d(kw)} \int_1^\infty e^{-kw\xi} \cdot \xi^{-3} d\xi$$

$$= -kdw \int_1^\infty e^{-kw\xi} \xi^{-2} d\xi = -kdw H_2(kw) \qquad (6)$$

and, correspondingly,

$$\int_0^W H_2(kw) kdw = H_3(0) - H_3(kW). \qquad (7)$$

Hence from (5)

$$S_{T_0}^W = 2\pi \int_0^\infty i_{\lambda T_0} H_3(0) d\lambda - 2\pi \int_0^\infty i_{\lambda T_0} H_3(k_\lambda W) d\lambda. \qquad (8)$$

The first term on the right (absorbing water vapor = 0) is the total black-body radiation σT_0^4, the second is the amount that the black-body surface transmits through layer W; see Eq. (2). For $w = \infty$ the latter term disappears, and thus it turns out that infinity on the w-scale in Fig. 6b is reached at σT_0^4 (4). The radiation transmitted by a black-body surface at temperature T through a vapor layer w is thus given by the area bounded by the abscissae w and ∞, the abscissa, and the curve T. This must be identical with the value that can be calculated from (2).

The final topic of this report is introduced in Fig. 7. The w scale clearly includes four orders of magnitude. In the case of gray radiation, infinity would be reached after two powers of ten; thus, in this case again the staggering of absorption is apparent. The bending of the curves for the individual temperatures can be explained roughly as follows: After transmission through 0.01 cm of water vapor, the maximum of the energy emerging lies at a wavelength between 20 to 25 μ; wavelengths with greater absorption coefficients have already
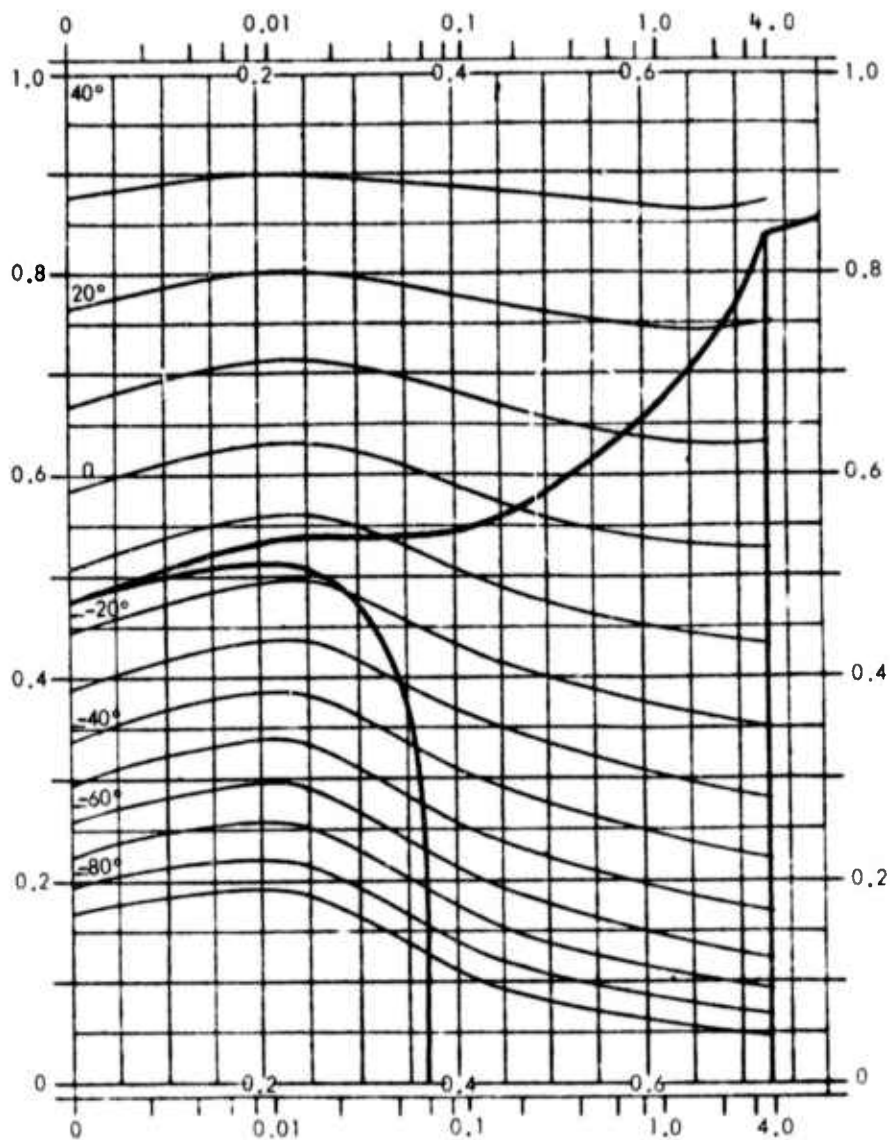

Fig. 7 -- Diagram for determining the radiation from the
water vapor and ground

been absorbed, while smaller k_λ still contain relatively small amounts of energy (Fig. 5). In the case of smaller w, the band from 6 to 7 μ also stands out quite noticeably, whereas with larger w the center of gravity of the energy gradually shifts from 20 to 10 μ. The wavelength, however, which after transmission through the vapor layer w furnishes the main part of the energy, also impresses its characteristic temperature dependence on the total energy, and thus the shift of the energy for w = 0.01 corresponds to that for large wavelengths--that is, only a slow decrease with decreasing T. At the beginning, the band from 6 to 7 μ is more effective; but at the end, the height of the T-lines reflects the temperature dependency of the Planck law at 10 μ.

In Fig. 7, a curve is plotted which shows the radiation through a horizontal area where a temperature of -15°C prevails. The upper part of the curve gives the radiation coming from below (from the warmer portion of the atmosphere), and also the radiation emitted by the ground, which has the same temperature as the immediately adjacent air layer. The lower part of the curve gives the radiation coming from above. The area between the two branches of the curve completely defines the net upward radiative flux, which is due to the radiation from the water vapor and the ground.

If one determines this total radiative flux S for various altitudes h, then its variation dS/dh is a measure of the amount of energy provided by the layer dh; that is, the energy emitted by the layer dh. Under the effect of this radiation, the layer undergoes the following change in temperature:

$$\frac{\partial T}{\partial t} = - \frac{1}{\rho c_p} \frac{dS}{dh} , \qquad (9)$$

which can be computed accordingly for various altitudes in the atmosphere.

To determine completely the temperature changes due to radiative influences, one must also determine the effect of direct solar radiation. The scattering of sunlight, which is slight in the long-wave portion of the solar spectrum anyway, can be disregarded, since heating is

due solely to absorption. F. E. Fowle [7] has studied individual bands
of the solar spectrum for quantities of water vapor ranging from 0.5
to 8 cm of precipitable water. By utilizing extra-terrestrial inten-
sities [8] and the values for ultraviolet and infra-red corrections [9]
provided by Fowle and Abbot, the energies absorbed by a water-vapor
layer of thickness w are determined as follows (Table 1, col. 2).
These numerical values can be fitted relatively easily to an analytical
expression that can be extrapolated both for small quantities of water
below 0.5 cm (high atmosphere) and for very large values (when the sun
is low). The formula $0.1720 \cdot w^{0.3028}$ yields the values in column 3
and the errors relative to the observed values (col. 4), which never reach
1 percent. For very large w > 8 cm, the values are certainly too large;
still this error is of no great significance, since large w occur only
very briefly when the sun is low.

Table 1

SOLAR RADIATION ABSORBED BY w cm OF PRECIPITABLE WATER

$$(\text{gcal} \cdot \text{cm}^{-2} \text{ min}^{-1})$$

w	From Fowle	Calculated	Error
0.5	0.1406	0.1394	-0.0012
1.0	0.1707	0.1720	+0.0013
1.5	0.1933	0.1944	+0.0011
2.0	0.2118	0.2121	+0.0003
3.0	0.2408	0.2399	-0.0009
4.0	0.2620	0.2617	-0.0003
5.0	0.2811	0.2800	-0.0011
6.0	0.2962	0.2959	-0.0003
7.0	0.3100	0.3100	0
8.0	0.3220	0.3228	+0.0008

The intensity impinging at a solar zenith angle z on the horizontal
area over which a quantity of water vapor w rests is given by

$$J = \left(J_0 - 0.1720 \left(\frac{w}{\cos z}\right)^{0.3028}\right) \cos z.$$

By differentiating with respect to w, the amount of energy absorbed
in the layer dw is obtained:

$$0.0521 \left(\frac{w}{\cos z}\right)^{-0.6972} dw.$$

The zenith angle z is a function of the time of day:

$$\cos z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos t = A + B \cos t,$$

where A and B are to be regarded as constants that vary only with the geographical latitude and the season [10]; t is the hour angle of the sun.

The solar energy absorbed over a 24-hour period is then given by

$$0.0521 \ w^{-0.6972} \ dw \cdot 2 \int_0^\tau (A + B \cos t)^{0.6972} \ dt, \tag{10}$$

where τ is defined by the time of sunrise: $\cos \tau = -A/B$. The above integral can be evaluated graphically for various latitudes and seasons and thus immediately shows what the heating power of solar radiation (Table 2) is under various conditions. It is worth noting that during the summer solstice the heating at the pole is 1.6 times that at the equator.

The heating resulting from absorption is obtained by division by ρc_p in (10).

Table 2

RATIO OF ABSORBED SOLAR ENERGY AVERAGED

OVER A 24-HOUR PERIOD TO THAT ABSORBED DURING VERTICAL INCIDENCE

δ	0°	30°	50°	65°	90°
+23 1/2°	0.330	0.401	0.424	0.448	0.527
+16 1/3°	0.340	0.380	0.372	0.359	0.412
0°	0.349	0.316	0.257	0.192	0
−16 1/3°	0.340	0.244	0.146	0.054	0
−23 1/2°	0.330	0.208	0.097	0.006	0

Figure 8 gives an example of the radiative effects. Batavia's vertical temperature distribution is chosen, and the relative humidity is assumed to be 50 percent throughout. Radiation from the atmosphere and

ground alone produces cooling at all altitudes; this is to be expected
as long as no solar radiation counterbalances the radiation lost to
space from the earth. In this case, the cooling exhibits a very pro-
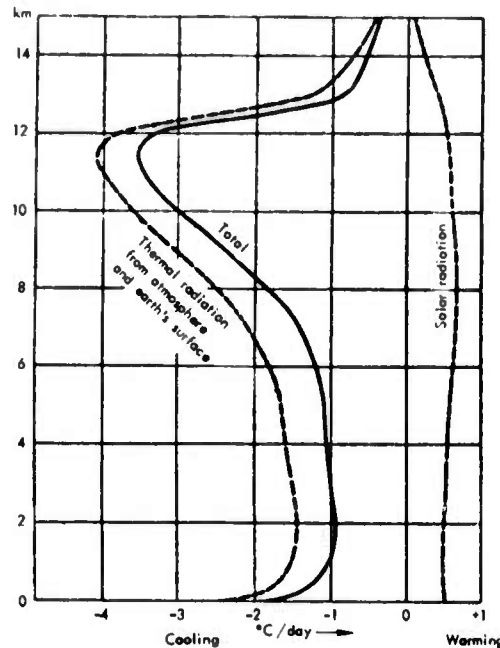nounced maximum at an altitude of 11.5 km, above which the effect

Fig. 8 -- Temperature change due to radiation in an atmosphere
using Batavic's temperatures

approaches zero very quickly. This radiation maximum has already
been noted by Albrecht, who calls it an emission layer; one should
note, however, that emission is not limited solely to this altitude,
but is quite important at lower altitudes as well. Heating due to
solar radiation (computed for $\varphi = 0°$, $\delta = 0°$) produces a rather uniform
and small value at all altitudes compared with the thermal radiation--
to such an extent that quite considerable cooling remains throughout
the atmosphere in the final analysis.

Opposing this is the effect of radiation at the earth's surface;
the effective radiation from the ground amounts to 0.0720, and the
incoming radiation from the sun to 0.5297 gcal cm^{-2} min^{-1} in the mean
throughout the day. In order to obtain values that correspond to

the actual mean relationships, the latter value should be reduced
by the amount of solar energy lost due to scattering and cloud cover
and augmented by the amount of diffuse sky radiation present. If
one takes this into account and then compares the quantity of heat
actually gained by the earth's surface to that given off by the atmo-
sphere, a surplus of energy is obtained that is not radiated back
into space at that location and must be carried off in some way,
since on the average warming cannot occur. On the contrary, in
making a comparison with the energy balance at other latitudes the
heating of the entire earth must equal zero in the mean. These
questions will be given closer scrutiny in an article in the Meteo-
rologische Zeitschrift.

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